Examinee's No. ________________

Department of Nuclear Engineering and Management
University of Tokyo
2007 Entrance Examination for the master course

Mathematics

August 28, 2006
13:00～15:30

Notice

1. Do not open this booklet until the start of the examination.
2. Write your examinee's number on the top of this sheet. Do not write your examinee's number or your name anywhere else.
3. There are 20 questions in total. Choose and answer 15 questions out of 20. **Circle the numbers of the questions you have selected in the bottom row.**
4. Write the outline of your solution below the question and put only the answer in the box.
5. Sheets for calculation are distributed separately.

<u>Circle the number of the questions selected. Notice that you can not answer more than 15 questions.</u>

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Q 1

Solve the differential equation

$$\frac{dy}{dx} + ay = \cos x$$

with the initial condition of $x = 0$, $y = 0$.

Here, a is a positive constant.

Q 2

Let

$$f(x) = \begin{cases} \dfrac{1}{e} & (x < 0) \\ e^{-x} & (0 \leq x) \end{cases}$$

and

$$F(x) = \int_{x-a}^{x} f(t)dt$$

.Find x which maximizes $F(x)$ and the maximum value of $F(x)$.
Here, e denotes the base natural logarithm and a is a positive constant.

Q 3

Sketch the graph of $f(x)= x^n e^{-x}$ (n is a fixed integer equal to 2 or greater than 2) for $0 \leq x \leq 3n$. In particular, indicate the characteristic points with quantitative information.

Q 4

Show all the couples of the positive natural numbers m and n which satisfy the following equation.

$mn - 3m - 2n = 0$

Q 5

Let $S(x) = \frac{x^4}{2\times 4} + \frac{x^6}{2\times 4\times 6} + \frac{x^8}{2\times 4\times 6\times 8} + \cdots$. Answer the following questions.

(1) Derive the first order differential equation regarding $S(x)$.

(2) Solve the above differential equation for $S(x)$.

Q 6

Let $S_N = \sum_{n=0}^{N} x^n$ and $x = i = \sqrt{-1}$. Derive the magnitude of S_N . Note that the magnitude of a complex number $z = a + bi$ is defined as $|z| = \sqrt{z\bar{z}} = \sqrt{a^2 + b^2}$.

Q 7

x is a random variable in the range $0 < x < 2m$ which obeys the probability distribution:

$$f(x) = -A(x^2 - 2mx)$$

Answer following questions. Here A, m is a positive constant.

(1) Find A which makes $f(x)$ be normalized.

(2) Prove that the expected value of x (population mean) is m.

(3) Find the expected value of $(x-m)^2$ (population variance).

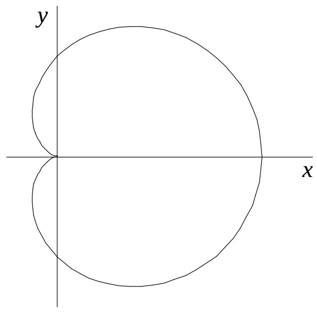
(4) Let $x_1, x_2, \cdots, x_n$ be random samples of the random variable x. Define a sample mean:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Prove that the expected value of $\bar{x}$ is also m.

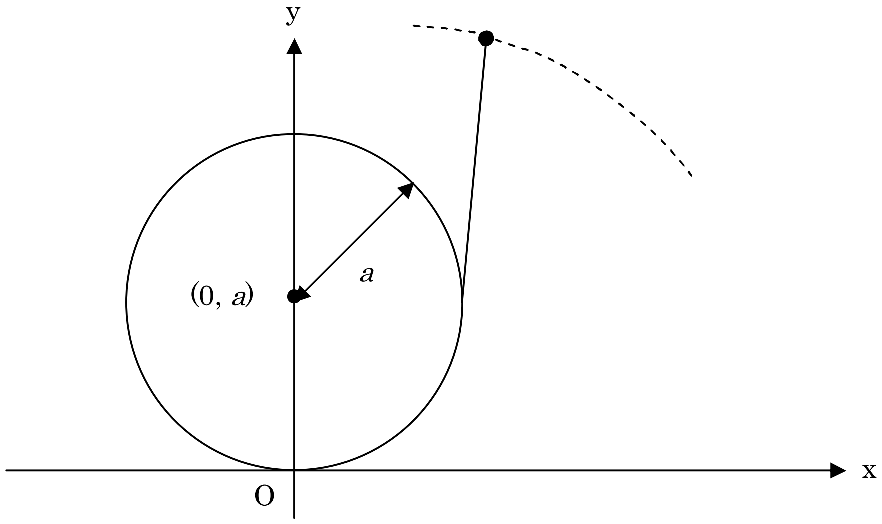
Q 8

The cardioid shown below is expressed by the polar equation r=1+cosθ. Find the surface area A generated by revolving it around the x-axis.



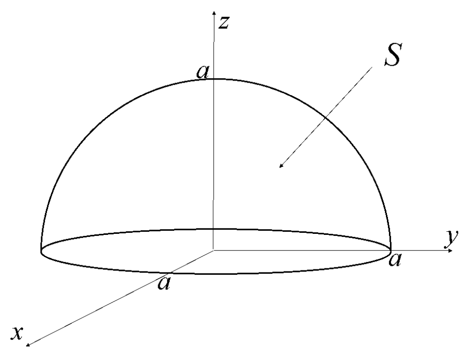
Q 9

Wrap a string of length $a\pi$ around a circle of radius a centered at $(0,a)$. One end of the string is tied to the origin $(0,0)$. What is the length of the trajectory of the string end? Consider just the first quadrant.



Q 10

Let $\boldsymbol{A} = \begin{pmatrix} -y + xz \\ x + yz \\ z + xy \end{pmatrix}$. Let S be the surface of the hemisphere given by $x^2 + y^2 + z^2 = a^2 \quad (z \geq 0)$. Calculate the surface integral; $\int_S (\nabla \times \mathbf{A}) \cdot n dS$. ($n$ is the normal vector of surface S)



Q 11

Answer the following questions.

(1) Find the ratio of the area of a circumscribed equiangular hexagon and the area of inscribed equiangular hexagon in a circle.

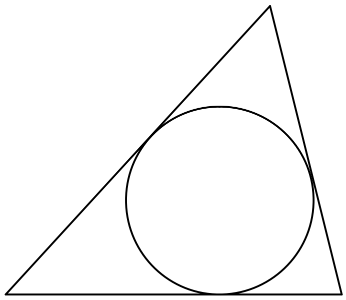
(2) Find the ratio of the areas of a circumscribed equiangular n polygon and an inscribed equiangular n polygon in a circle.

Q 12

Derive the volume of the region which satisfies both equations $x^2 + y^2 + z^2 \le 9$ and $3x^2 + 3y^2 - z^2 - 6z - 9 \le 0$.

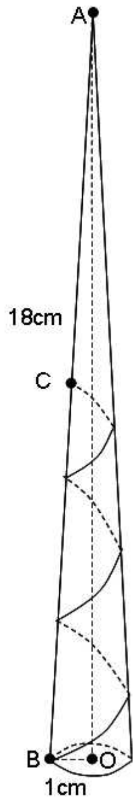
Q 13

Suppose that a circle (area=A) is circumscribed within a triangle (area=B). Derive the total length of the three sides of this triangle.



Q 14

There is a circular cone in which the length of the generating line is 18cm and the radius of the base circle is 1cm. Point C is a midpoint between points A and B. Calculate the length of the shortest distance from point B to point C, going round the side of the circular cone three times.



Q 15

Insert positive natural numbers, which are different to each other, in the table. These nine numbers must be selected so that the products of three natural numbers on each row, on each column, and on each diagonal line are identical in value. Then calculate the value of x.

4	x	
		8
64		

Q 16

Given the calculation below, obtain A×D – B×C. A, B, C, and D are natural numbers between 1 and 9.

$$
\begin{array}{r}
B\,C\,A\,C \\
+\,B\,C\,D\,D \\
\hline
C\,C\,B\,A
\end{array}
$$

Q 17

The cryptogram「○□△☆×」 is decoded into「970」, 「△□△☆×」into 「2220」and 「☆△□○×」 into 「2930」. Under the same rule, what is the cryptogram for 「781」?

Q 18

Suppose that a rectangular is divided into sub-regions by n straight lines. Derive the maximum number of sub-regions.

Q 19

From five people of A, B, C, D and E, there are only two honest people who tell the truth all the time. The other three are liars who say a mixture of truth and false.
Find the two honest people based on the answers below to the question ‘who is a liar’. Also describe your process of reasoning.

A: “Neither C nor D is a liar.”
B: “C is a liar.”
C: “D is a liar.”
D: “E is a liar.”
E: “Both B and C are liars.”

Q 20

Find out the value of (A) in the following series?

64→28→68→76→50→(A)→2→4→16→38→70